Calculus: Questions 2
Lagrange Multipliers

In all the following use Lagrange multiplers to incorporate the constraints into the problems, even if it is quicker to do it in another way.

1. Find the extremum (maximum or minimum) of
$$x^2 - 2y^2 + 2xy + 4x$$
with the constraint $2x = y$

2. What are the extrema of
$$f(x, y) = x + x^2 + 4y$$
on the circle $x^2 + x + y^2 + 2y = 1$?

3. What point on the spere
$$x^2 + y^2 + z^2 = 1$$
is closest to the point $x = 1$, $y = 2$, $z = 3$?

4. Find local maxima and minima in the distance from the origin to the curve
$$x^3 + y^3 - 6xy = 0.$$

5. You want to make an opened top box with volume 4. You are required to use as little material as possible in making it. Find the dimensions of the box so that the area of the box is minimised.

6. (Some problems don't work out. Try this and see what happens.) You have boarding to make a compost heap. This requires you to make 4 perpendicular walls that will be placed directly on the ground. If you have $4\,\mathrm{m}^2$ of boarding, find the maximum volume of compost that your heap can contain. Assume that you will not fill the heap any higher than the top of the walls.

Solutions

1. $x = 2/3$, $y = 4/3$, $f = 4/3$
2. $x = -1/2$, $y = 1/2$, $f = 7/4$
   $x = -1/2$, $y = -5/2$, $f = -41/4$
3. $x = 1/\sqrt{14}$, $x = 2/\sqrt{14}$, $z = 3/\sqrt{14}$
4. Minima at $x = y = 0$ (two of them if you check the curve!). Maximum at $x = y = 3$
5. Height 1, with a square base of sides 2.
6. You will not find a sensible maximum or minimum. If compost heap is square, and each wall has area 1, then each side will have height h and width $1/h$. The volume is $h \times (1/h) \times (1/h) = 1/h \to \infty$ as $h \to 0$.