Calculus: Questions 7
Differential Equations

These questions are mostly revision of last year's material.

1. Which of the following are exact differential equations (i.e., of the form $\frac{dF}{dx} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{dy}{dx} = 0$)

   (a) $3x^2 + y^3 + 3xy^2\dfrac{dy}{dx} = 0$ (b) $2x^2 + y^3 + 2xy^2\dfrac{dy}{dx} = 0$

   (c) $x\cos y - x^2 \sin y\dfrac{dy}{dx} = 0$ (d) $x\cos^2 y - x^2\cos y \sin y\dfrac{dy}{dx} = 0$

2. For the answers to question 1 that are exact differential equations, integrate to find the solution (i.e., find $F = C$)

3. Use integrating factors to find the solutions to

   (a) $\dfrac{dy}{dx} + x^3 y = 0$ (b) $\dfrac{dy}{dx} - y\tan x = \sin^2 x$

4. Use separation of variables to find solutions to

   (a) $\dfrac{dy}{dx} + x^3 y = 0$ (b) $\dfrac{dy}{dx} - y\cot x = 0$

5. Use change of variable to find solutions to

   (a) $x^3\dfrac{dy}{dx} = yx^2 - y^3$ (b) $(x^3 + xy^2)\dfrac{dy}{dx} = y^3$

6. Find the general solutions to the following differential equations:

   (a) $\dfrac{d^2y}{dx^2} - y = e^{3x}$

   (b) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{-2x}$

Solutions

1. (i) yes, (ii) no, (iii) no, (iv) yes.

2. (a) $x^3 + xy^3 = C$ (b) —

   (c) — (d) $\frac{1}{2}x^2 \cos^2 y = C$

3. (a) $R(x) = e^{x^4/4}, \quad y = Ce^{-x^4/4}$ (b) $R(x) = \cos x, \quad y = \frac{\sin^3 x}{3\cos x}x + \frac{C}{\cos x}$

4. (a) $y = Ce^{-x^4/4}$ (b) $y = C\sin x$

5. (a) $x = C\exp(x^2/(2y^2))$ (b) $y = C\exp(-y^2/(2x^2))$

6. (a) $y = Ae^x + Be^{-x} + \frac{e^{3x}}{8}$

   (b) $y = Ae^{-x} + Be^{-2x} - xe^{-2x}$