Throughout the course, $\mathbb{F}$ will denote either $\mathbb{R}$ (the set of real numbers) or $\mathbb{C}$ (the set of complex numbers).

Definition 1.1
A **vector space over** $\mathbb{F}$ is a set V together with two operations

- an *addition*, denoted $\oplus$, which assigns to each pair of elements $\mathbf{u}, \mathbf{v} \in V$, an element $\mathbf{u} \oplus \mathbf{v} \in V$;
- a *scalar multiplication*, denoted $\cdot$, which assigns to each element $\mathbf{v} \in V$ and each scalar $\lambda \in \mathbb{F}$, an element $\lambda \cdot \mathbf{v}$;

satisfying the following properties:

(V1) $\mathbf{u} \oplus (\mathbf{v} \oplus \mathbf{w}) = (\mathbf{u} \oplus \mathbf{v}) \oplus \mathbf{w}$ for all $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$

(V2) $\mathbf{u} \oplus \mathbf{v} = \mathbf{v} \oplus \mathbf{u}$ for all $\mathbf{u}, \mathbf{v} \in V$

(V3) There exists $\mathbf{0} \in V$ such that $\mathbf{v} \oplus \mathbf{0} = \mathbf{v}$ for all $\mathbf{v} \in V$

(V4) For all $\mathbf{v} \in V$, there exists some $(-\mathbf{v}) \in V$ such that $\mathbf{v} \oplus (-\mathbf{v}) = \mathbf{0}$.

(V5) $\lambda \cdot (\mathbf{u} \oplus \mathbf{v}) = (\lambda \cdot \mathbf{u}) \oplus (\lambda \cdot \mathbf{v})$ for all $\mathbf{u}, \mathbf{v} \in V$ and all $\lambda \in \mathbb{F}$

(V6) $(\lambda + \mu) \cdot \mathbf{v} = (\lambda \cdot \mathbf{v}) \oplus (\mu \cdot \mathbf{v})$ for all $\mathbf{v} \in V$ and all $\lambda, \mu \in \mathbb{F}$

(V7) $\lambda \cdot (\mu \cdot \mathbf{v}) = (\lambda\mu) \cdot \mathbf{v}$ for all $\mathbf{v} \in V$ and all $\lambda, \mu \in \mathbb{F}$

(V8) $1 \cdot \mathbf{v} = \mathbf{v}$ for all $\mathbf{v} \in V$.