Linear Algebra: Exercise Sheet 2

1. Which of the following sets are linearly independent, which ones are spanning sets for $\mathbb{R}^3$ and which ones form a basis for $\mathbb{R}^3$?
   (a) $\{(1,0,0)\,,\,(5,5,0)\,,\,(3,3,3)\}$
   (b) $\{(2,5,4)\,,\,(-2,1,0)\}$
   (c) $\{(-3,2,1)\,,\,(0,0,0)\,,\,(1,2,-3)\}$
   (d) $\{(1,1,1)\,,\,(6,2,4)\,,\,(7,3,5)\}$
   (e) $\{(1,0,0),(1,1,0),(1,1,1),(-1,5,2)\}$
   (f) $\{(1,0,0),(2,0,0),(3,0,0),(4,0,0)\}$

2. Which of the following sets are linearly independent, which ones are spanning sets for P_2 and which ones form a basis for P_2?
   (a) $\{1\,,\,1+x\,,\,1+x+x^2\}$
   (b) $\{3+x-x^2\,,\,2+2x+x^2\,,\,-1+x+2x^2\}$
   (c) $\{1+x+x^2\,,\,1\,,\,x\,,\,14x^2\}$

3. Let f,g,h be elements of the real vector space $\mathbb{R}^{\mathbb{R}}$ given by $f(x)=\sin x$, $g(x)=\cos x$, $h(x)=x$. Show that $\{f,g,h\}$ is a linearly independent set. What is the dimension of the subspace of $\mathbb{R}^{\mathbb{R}}$ spanned by f,g and h?

4. Find a basis for the subspace U of P_n consisting of all polynomials $p(x)\in P_n$ satisfying $p(0)=0$. What is the dimension of U?

5. Find a basis for the vector space $V=\{(x,y,z,t)\in\mathbb{R}^4 \mid x+y=z,\ x-y=t\}$ and hence find the dimension of V.

6. **(optional)** Show that the vector space P of *all* polynomials in one variable is infinite dimensional, i.e. show that it cannot be spanned by a finite number of vectors.