Linear Algebra: Solutions to January 2006 Exam paper

This is only a very rough sketch of the solutions to this exam paper, giving mainly the final answers and directing you to the corresponding questions in the exercise sheets for the method used.

1. (a)   i. This is a subspace. Check the three conditions as in Exercise Sheet 1, questions 4,5,6,7.

      ii. This is not a subspace (for example, condition (S1) fails).

     iii. This is a subspace. Check the three conditions as in Exercise Sheet 1. questions 4,5,6,7.

  (b) Take for example $\{(1,0),(0,1)\}$ and $\{(1,0),(1,1)\}$.

  (c)   i. Not linearly independent but spanning, so not a basis (as in Exercise Sheet 2, questions 2,3).

      ii. Linearly independent and spanning, hence a basis (as in Exercise Sheet 2, questions 2,3).

2. (a)   i. f is linear. Check two conditions as in Exercise Sheet 3, question 1.

      ii. f is not linear (for example $f(-1(1,1)) \neq -1f(1,1)$).

     iii. f is linear. Check two conditions as in Exercise Sheet 3, question 1.

  (b) No, as $f(2(1,1)) \neq 2f(1,1)$.

  (c) Bookwork.

  (d) (as in Exercise Sheet 3, question 5) $\operatorname{Ker} f = \{(-y,y,-y) \mid y \in \mathbb{R}\}$ with basis $\{(-1,1,-1)\}$ (there are other bases).
$\operatorname{Im} f = \{(x,0,z) \mid x,z \in \mathbb{R}\}$ with basis $\{(1,0,0),(0,0,1)\}$ (there are other bases).

3. (a) Bookwork.

  (b) Bookwork.

  (c) The matrix A has eigenvalues $\lambda = -2$ and $\lambda = 4$. Get matrix P with $P^{-1}AP$ diagonal with diagonal entries given by $-2,-2,4$ (in some order). (as in Exercise Sheet 5, question 3).

4. (a) Bookwork. $||\mathbf{x}|| = \sqrt{39}$.

  (b) Bookwork. The two given vectors are not orthogonal.

  (c) Bookwork. The given set of vectors is orthonormal (as in Exercise Sheet 6, question 4).

  (d) Applying Gram-Schmidt we get $\{\mathbf{v_1} = (1,0,0,0), \mathbf{v_2} = (0,\frac{1}{\sqrt{2}},0,\frac{1}{\sqrt{2}}), \mathbf{v_3} = (0,0,1,0), \mathbf{v_4} = (0,\frac{1}{\sqrt{2}},0,-\frac{1}{\sqrt{2}})\}$. (as in Exercise Sheet 7, question 2).