Fluid Dynamics

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The average density in a volume is

$$\text{density} = \rho = \frac{\text{mass}}{\text{volume}}$$

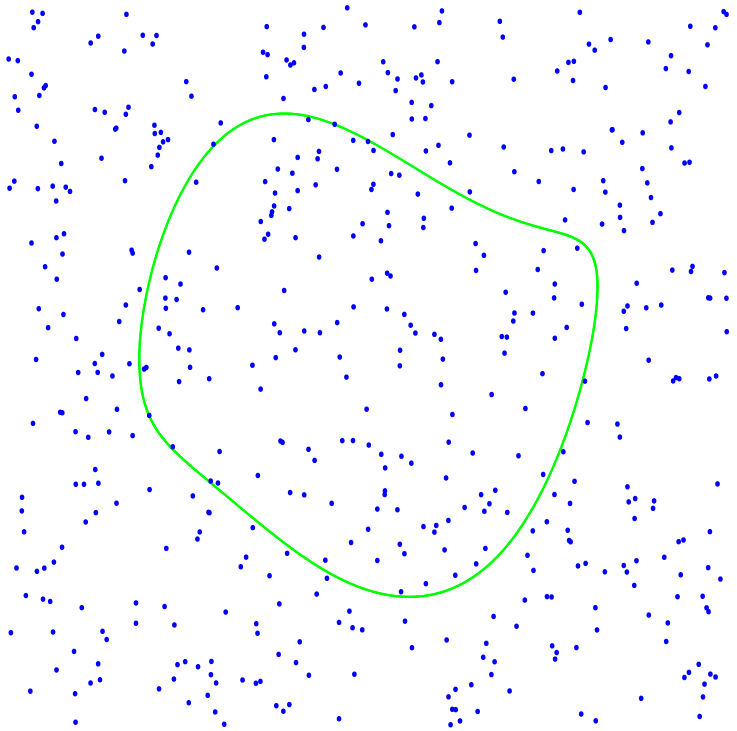
For a fluid we can also define a local density

$$\rho(\mathbf{x}, t)$$

Density can depend on position and time.

How to change the mass inside a volume

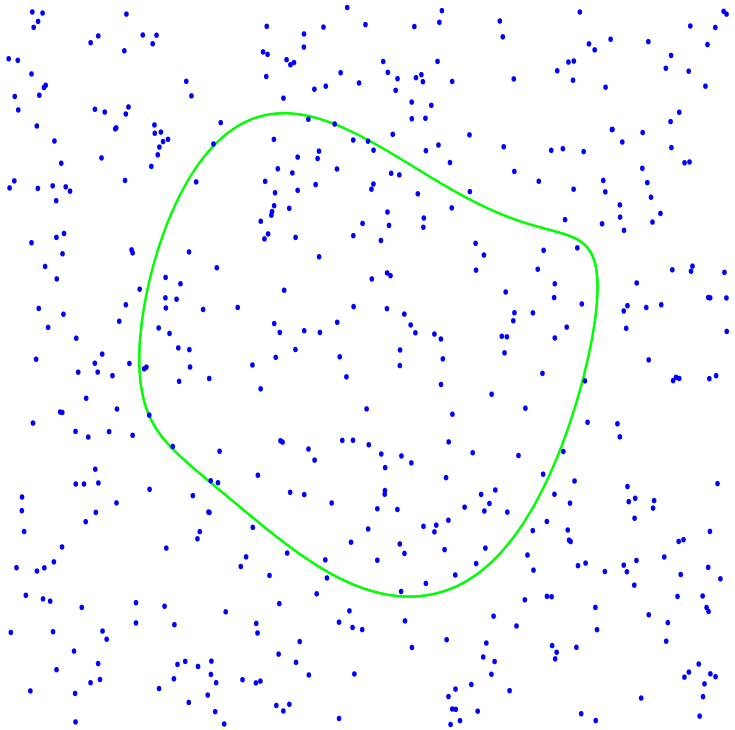
Consider a volume V fixed in space...



There are two ways you can change the mass of fluid inside this volume:

- Create/destroy mass
- Move mass across the boundary

Mass can be created



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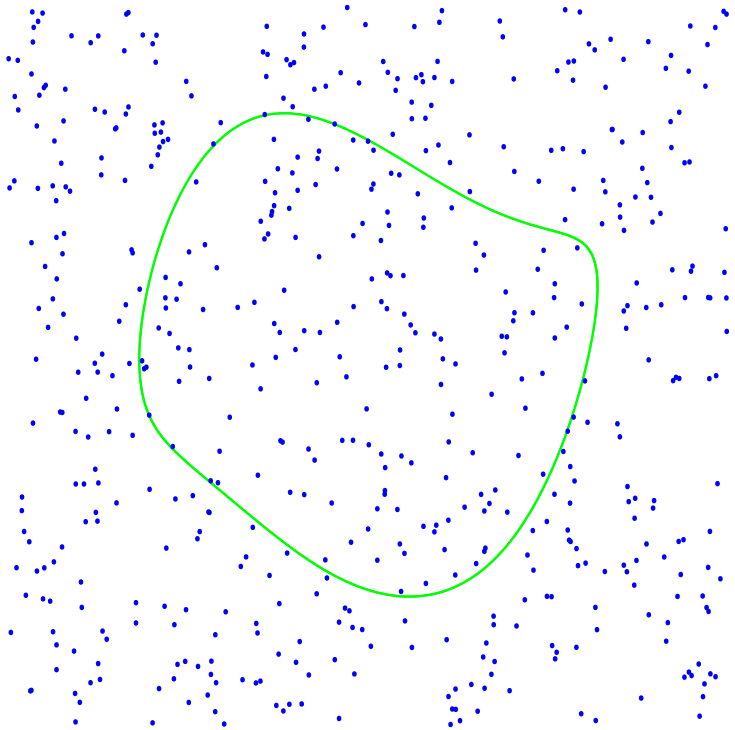

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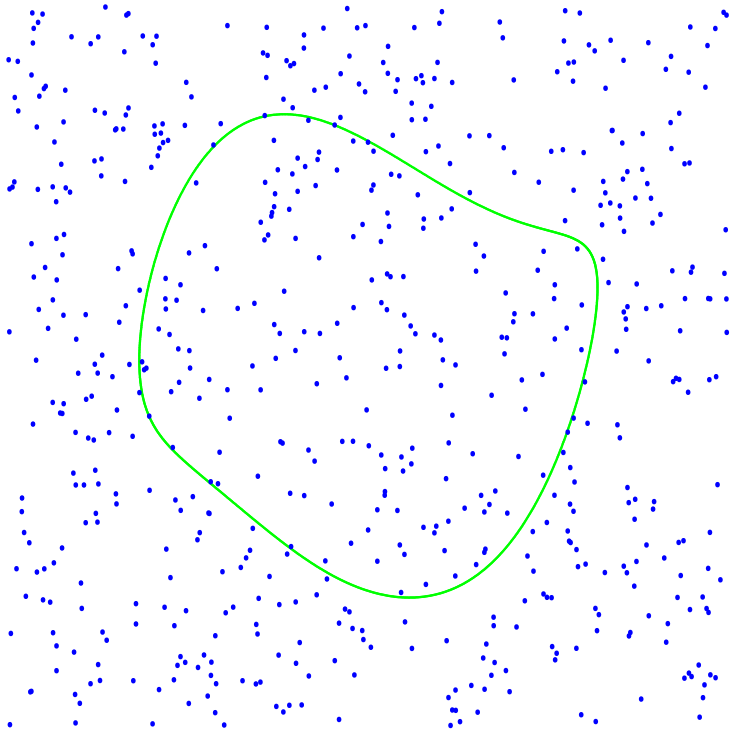

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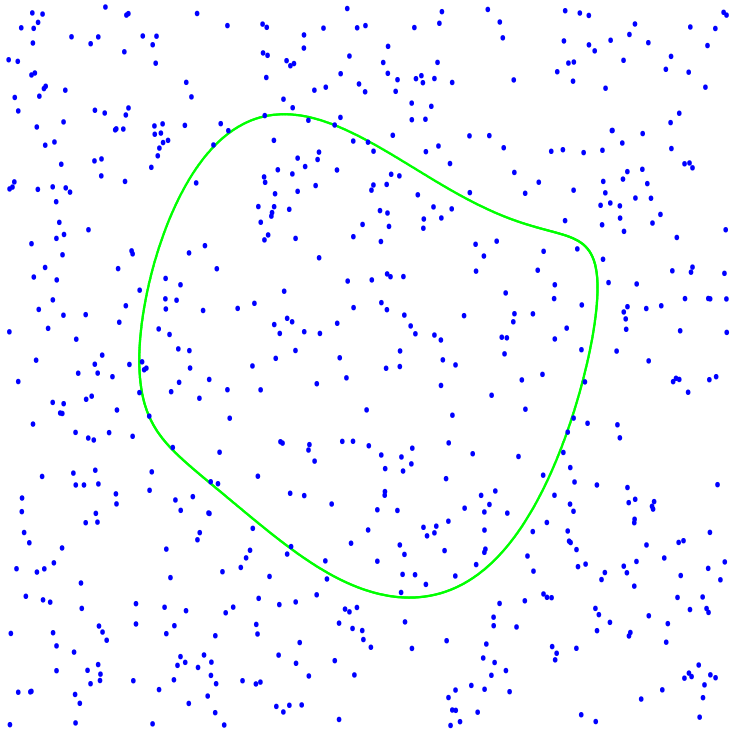

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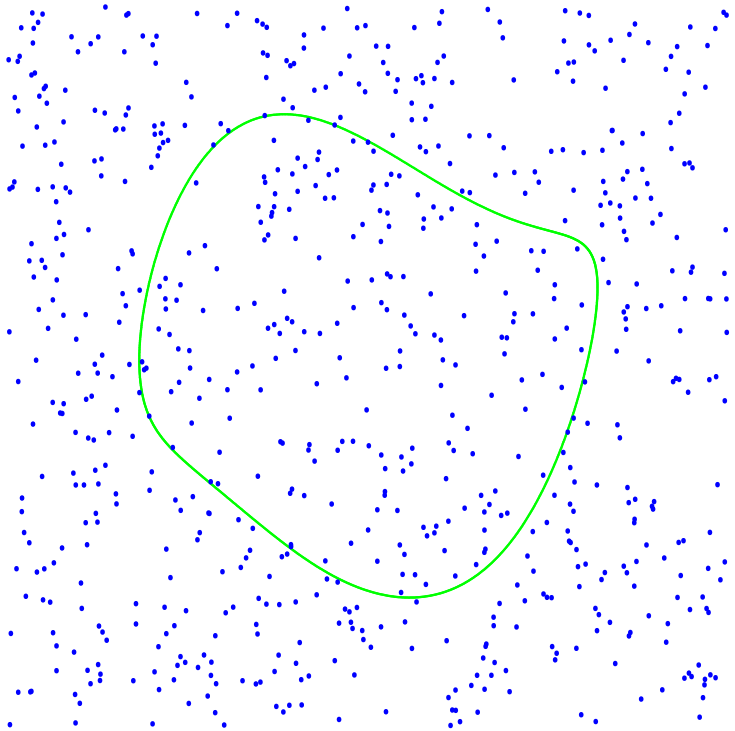

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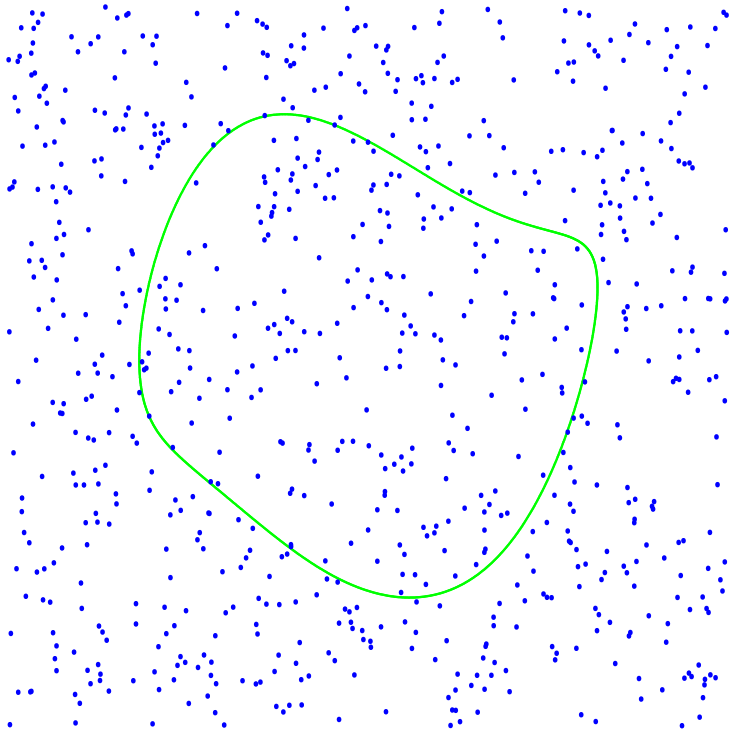

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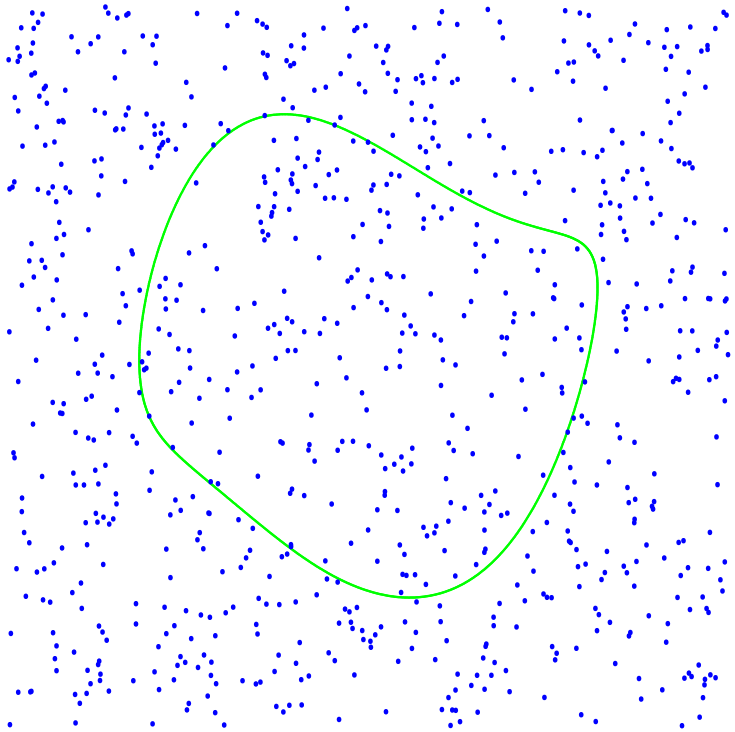

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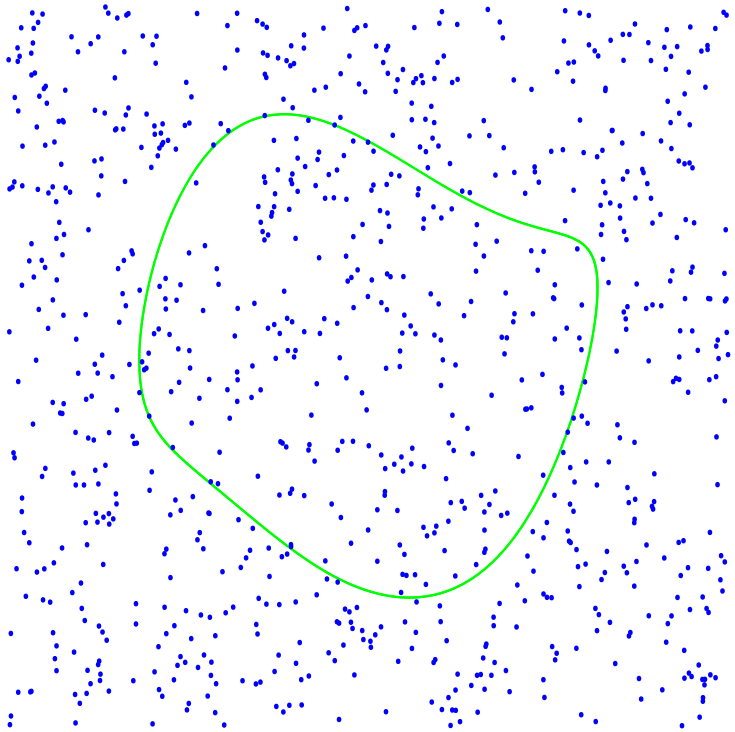

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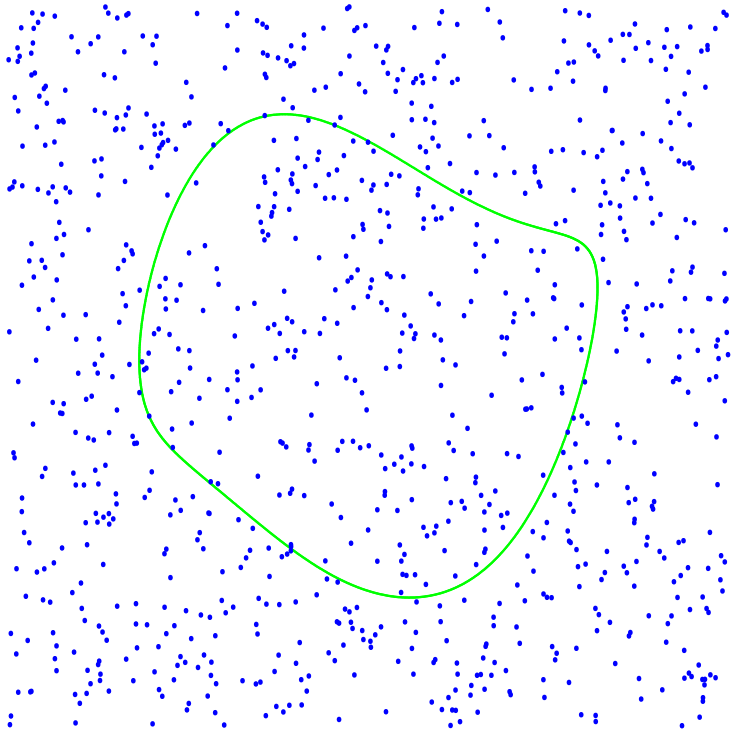

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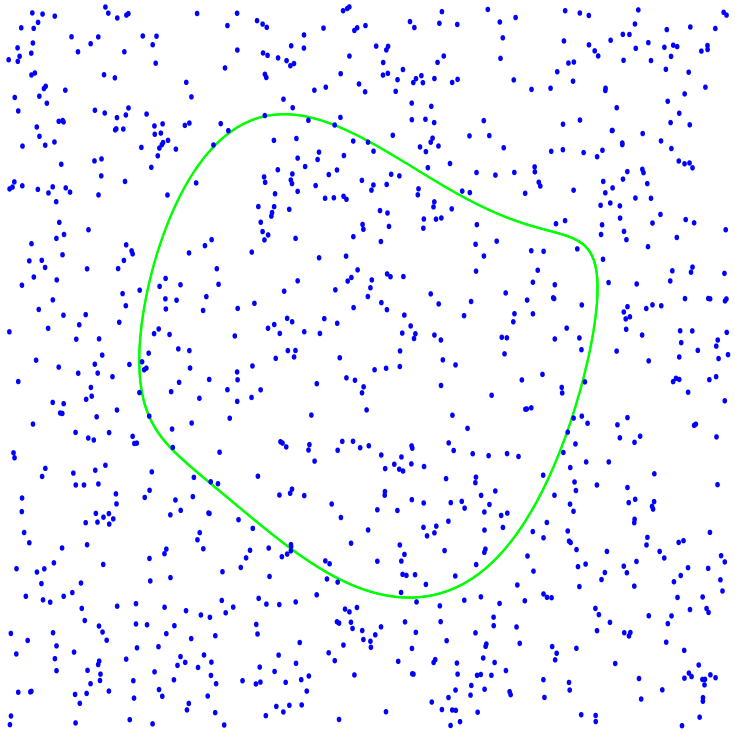

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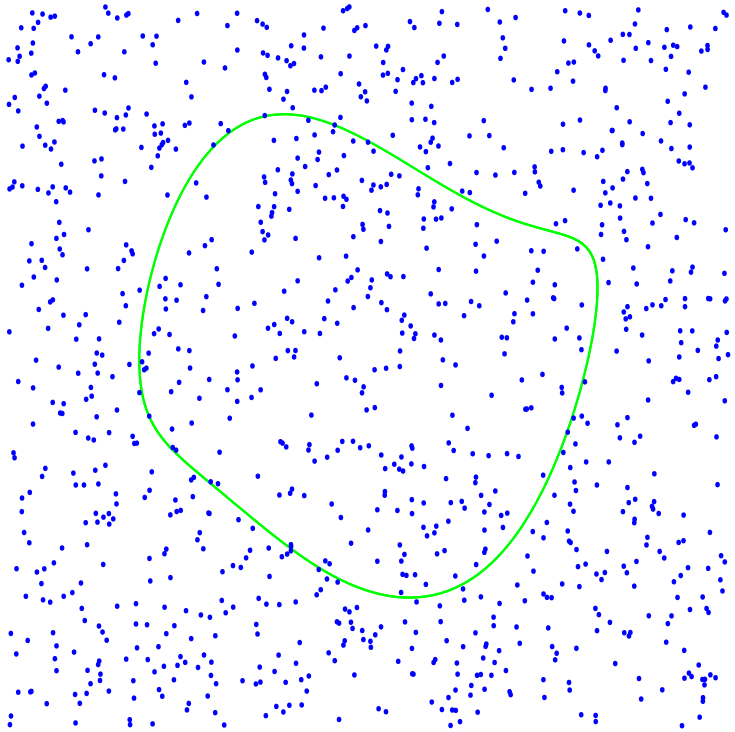

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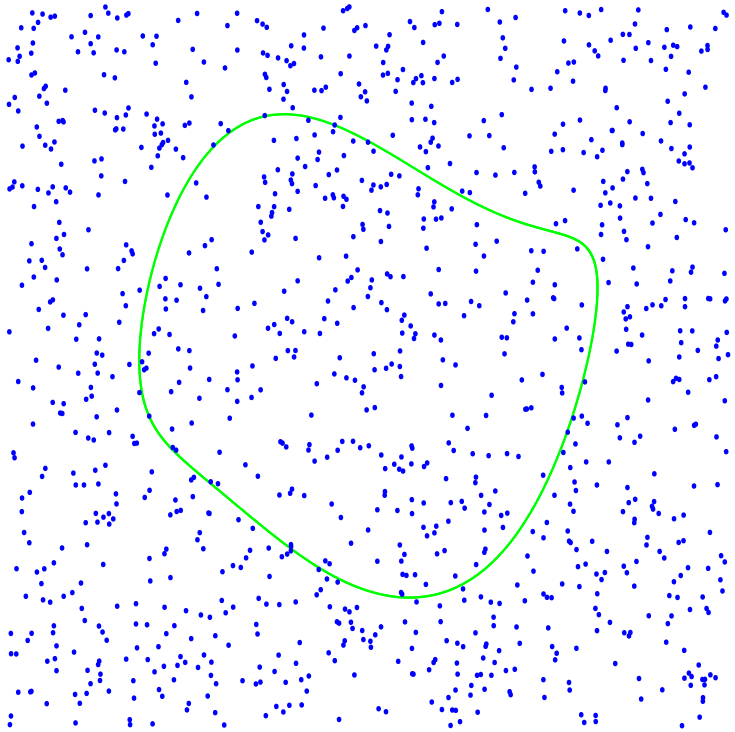

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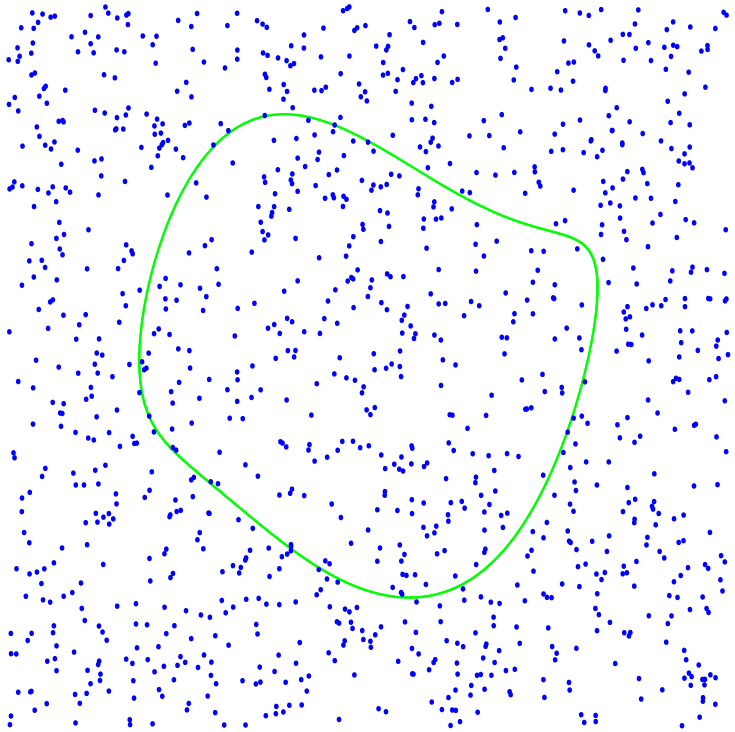

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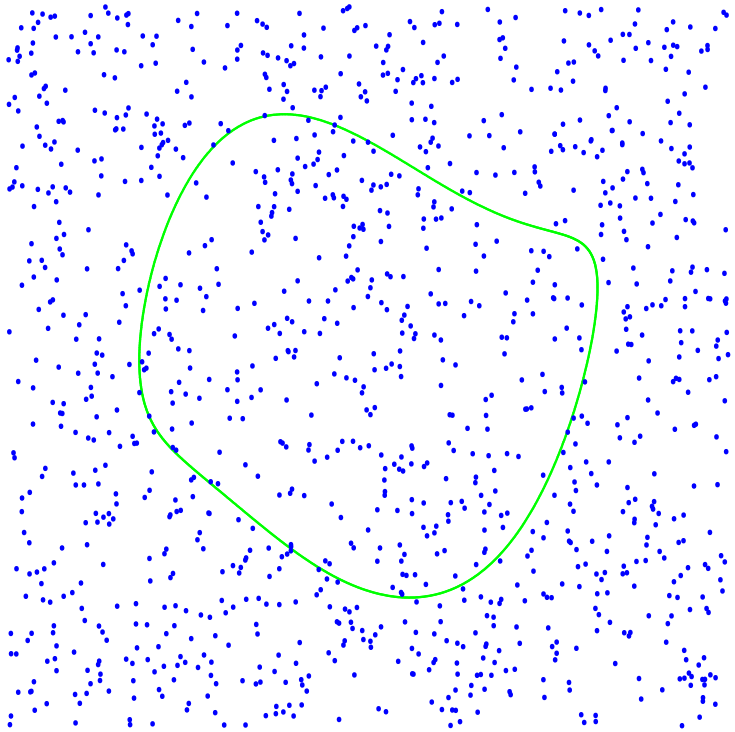

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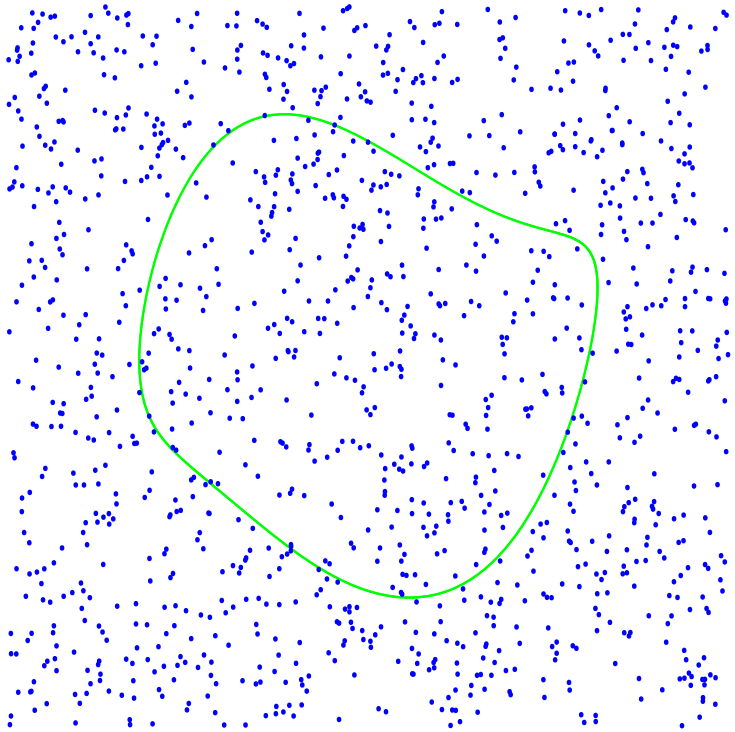

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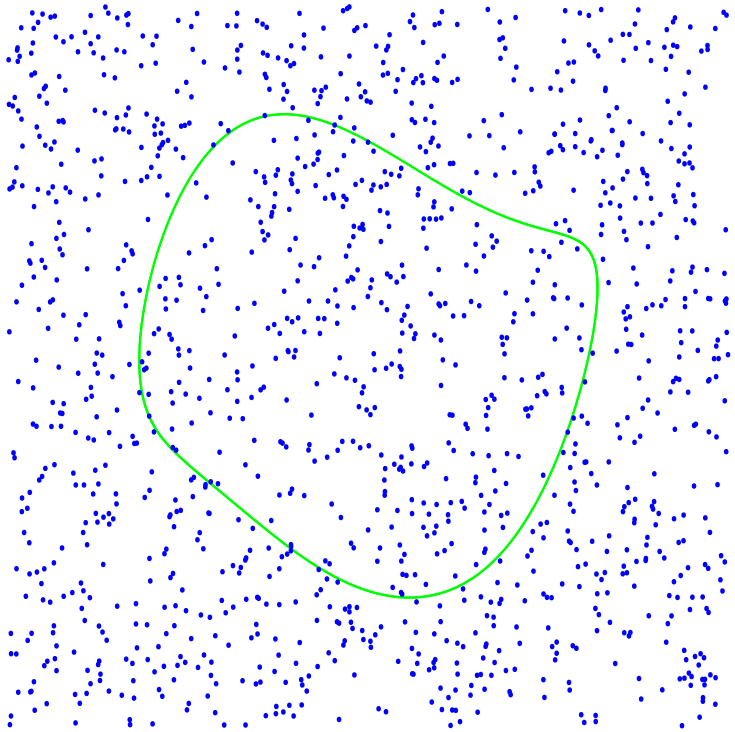

Problem

How do you create or destroy mass?

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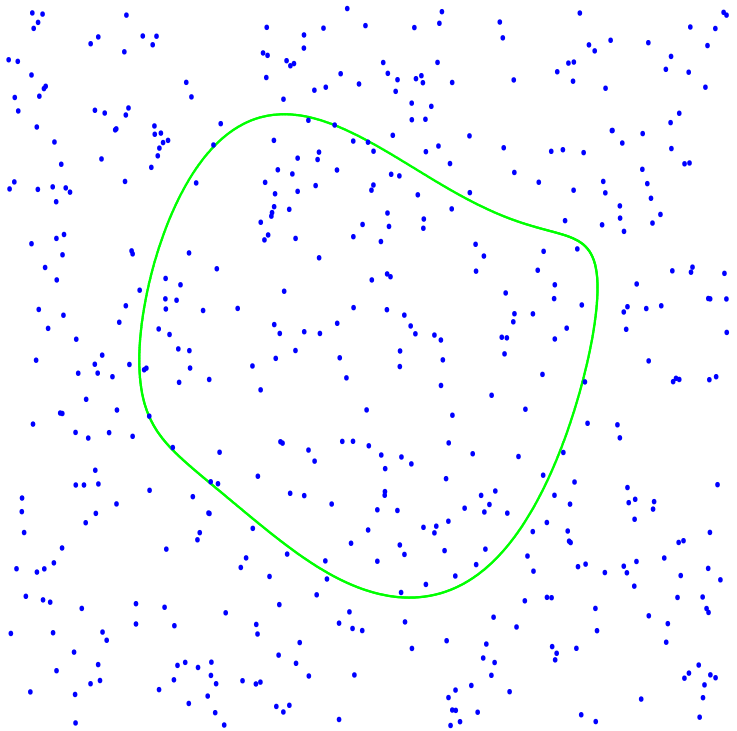
(may be a useful model)

Mass can move across the boundary



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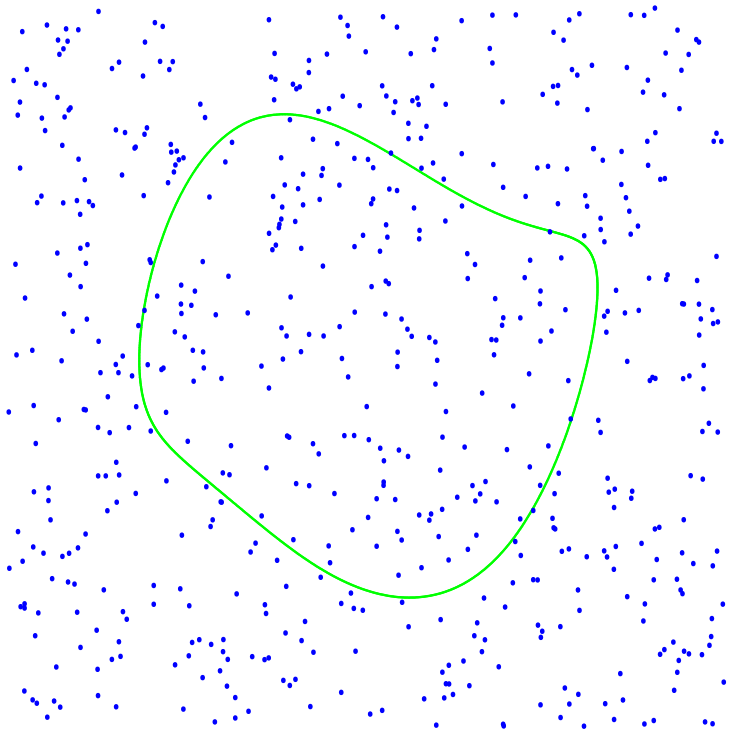

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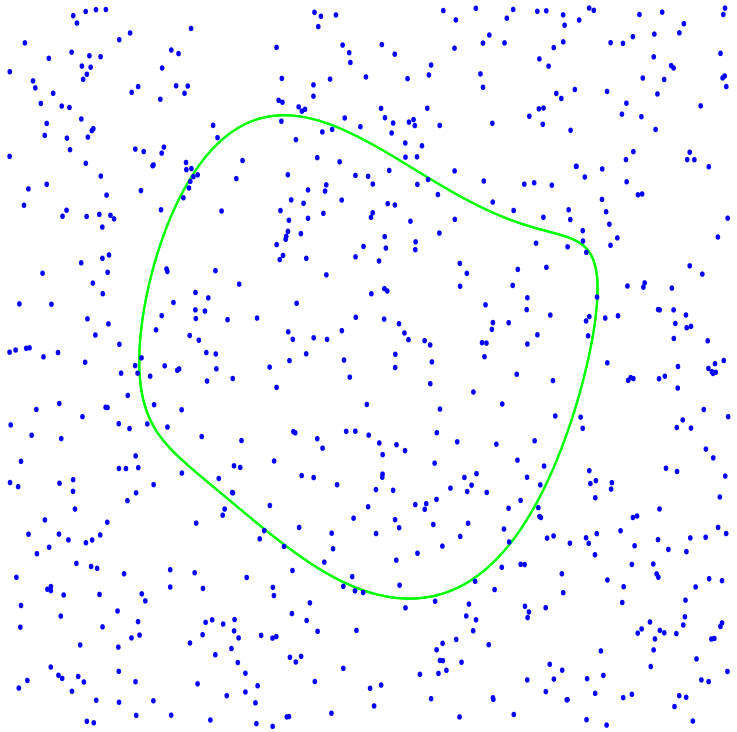

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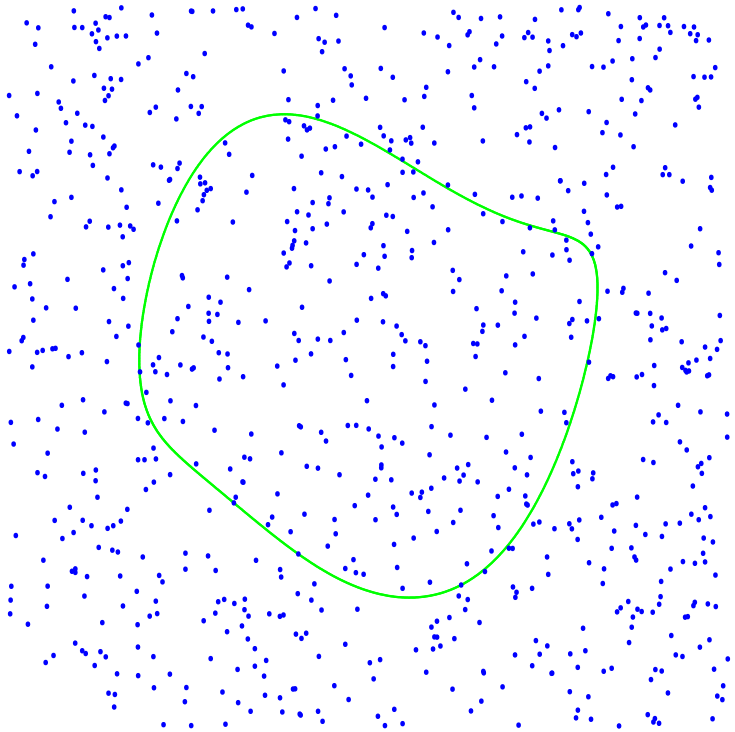

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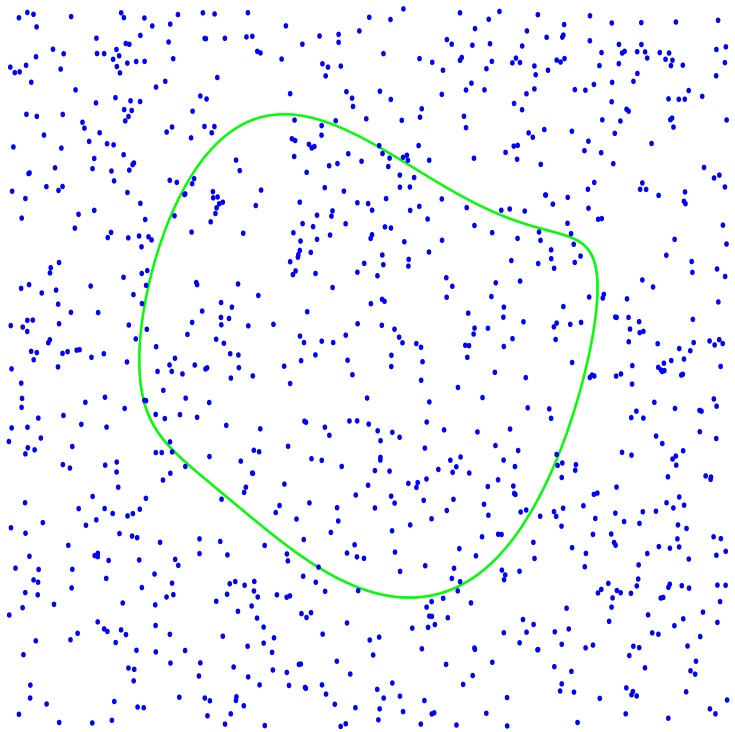

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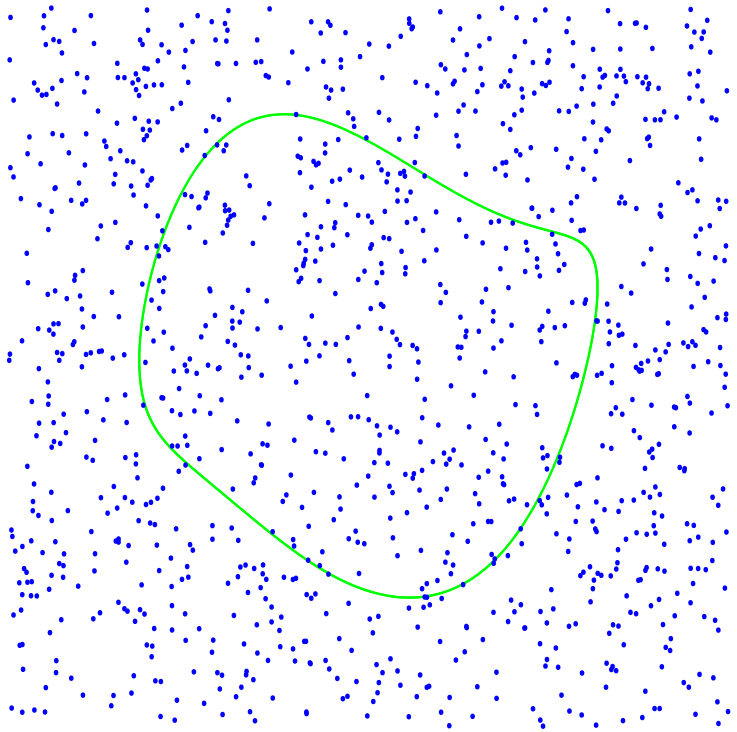

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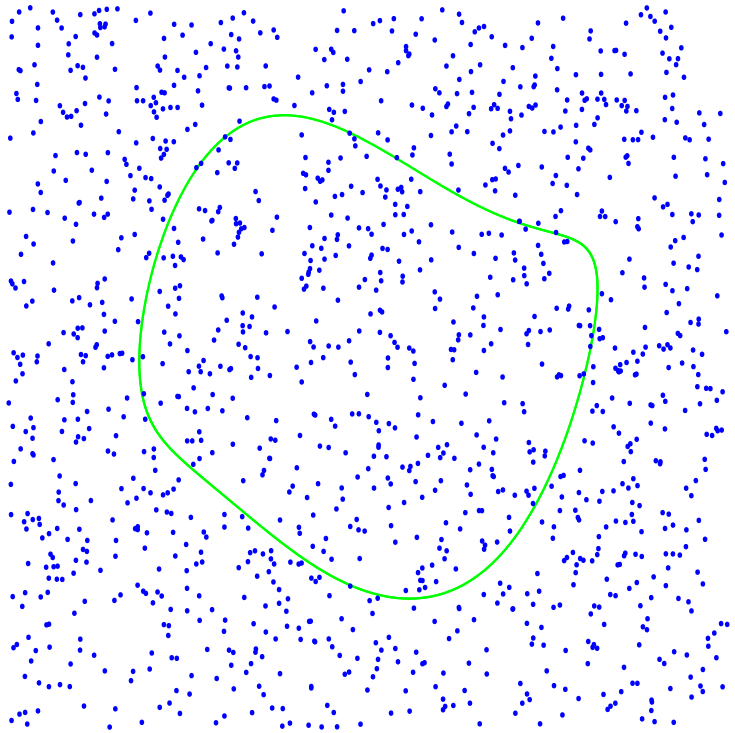

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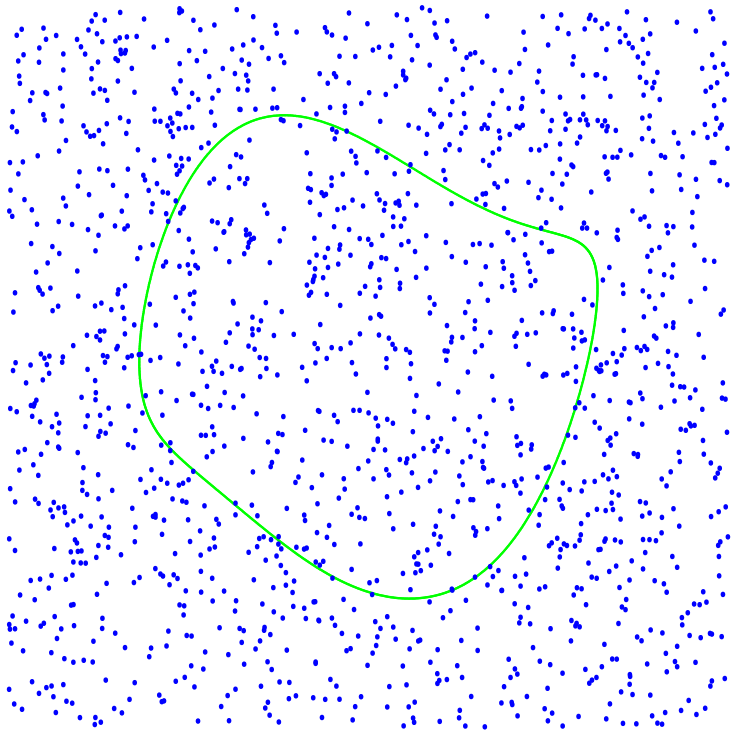

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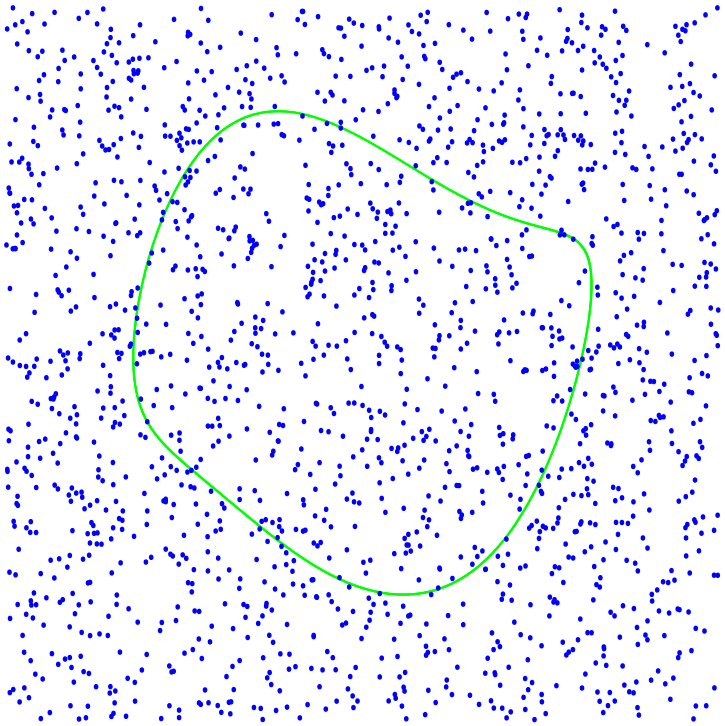

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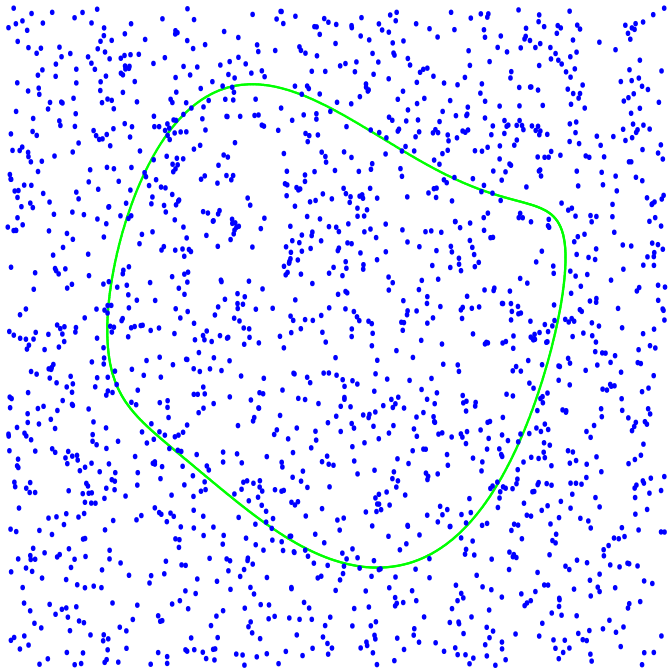

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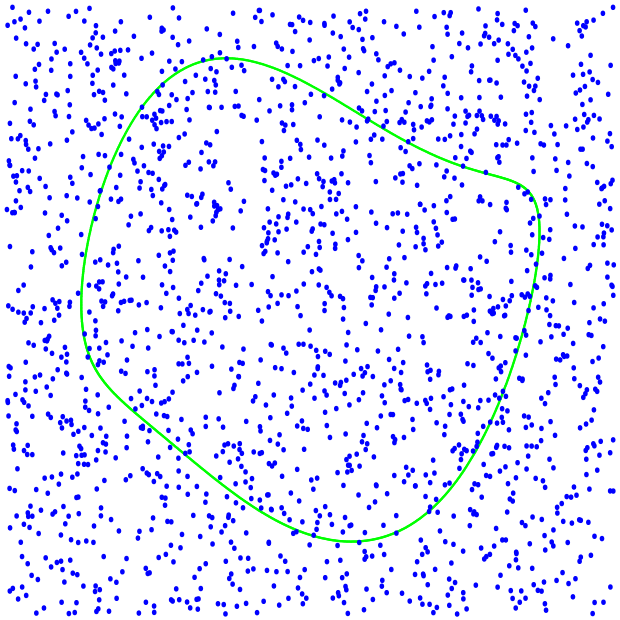
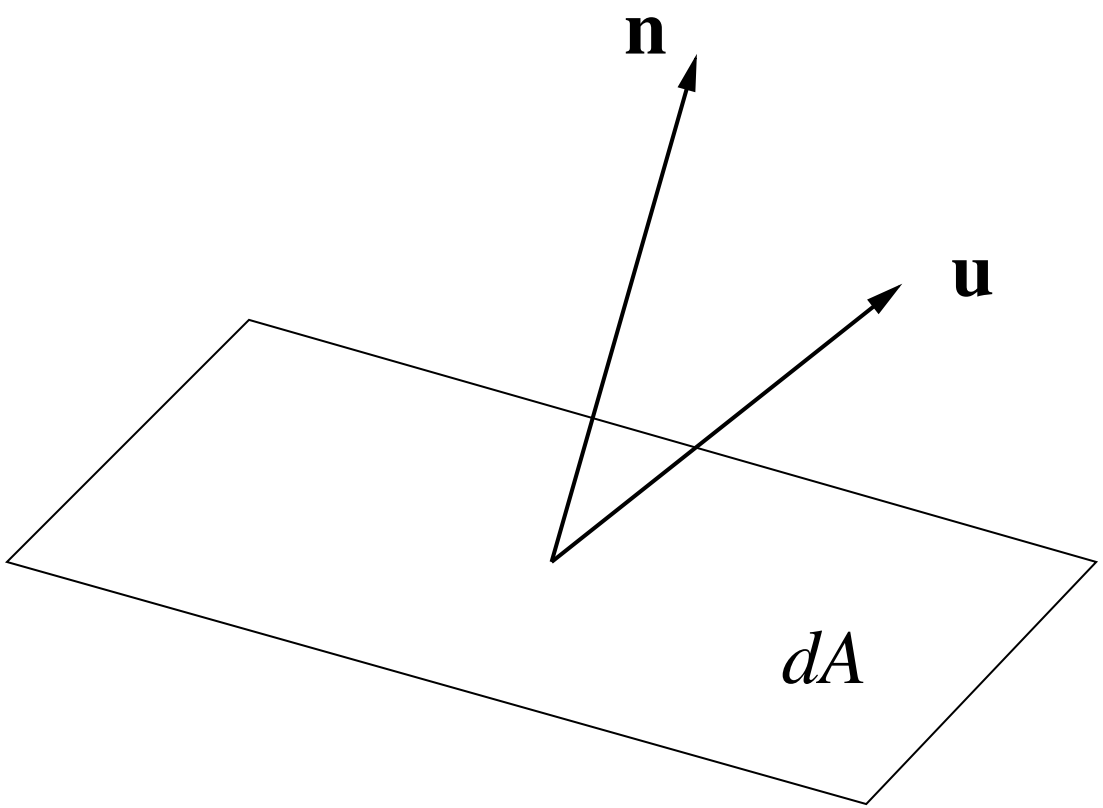


Mass of fluid, M, inside a boundary, V, is given by

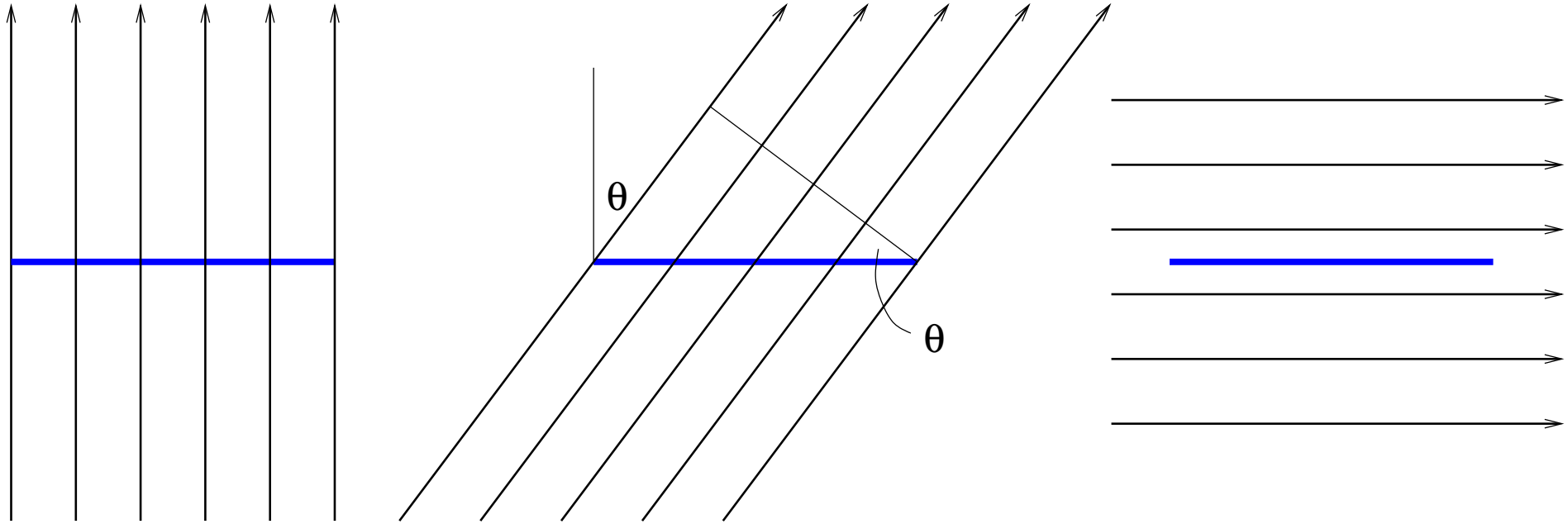
$$M = \int\int\int_V \rho \, dV$$

where $\rho(\mathbf{x}, t)$ is the density (mass per unit volume).

Flow across an area element

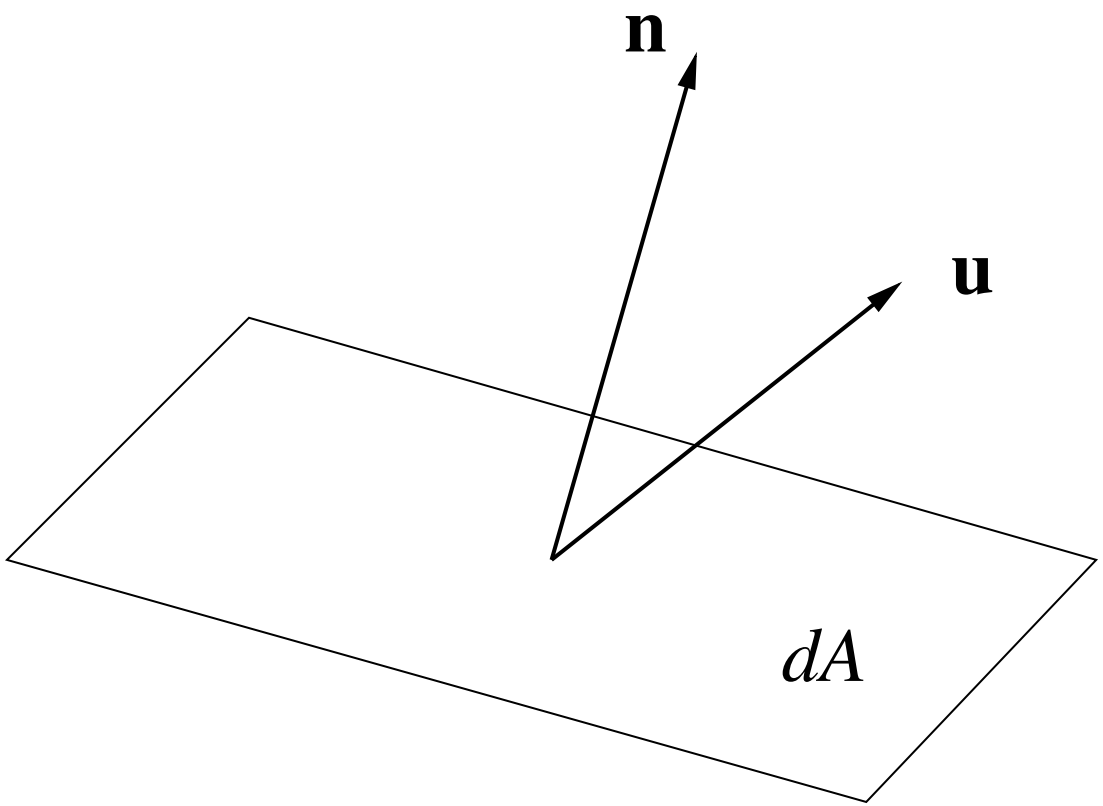


volume flow across element $= \mathbf{u} \cdot \mathbf{n}\, dA$



Width of stream crossing line is proportional to $\cos\theta$.
Flow across line is also proportional to the speed of flow, $|\mathbf{u}|$, and the width of the line, L.
So flow proportional to $|\mathbf{u}|L\cos\theta = \mathbf{u}\cdot\mathbf{n}\,L$ where $\mathbf{n}$ is normal to the line.

Mass flow across an area element



$$\text{mass flow across element} = \rho \times (\text{volume flow}) = \rho\, \mathbf{u} \cdot \mathbf{n}\, dA$$

$$\frac{dM}{dt} = \frac{d}{dt}\int\int\int_V \rho \, dV = -\int\int_S \rho \, \mathbf{u} \cdot \mathbf{n} \, dA$$

where S is the surface of the closed volume V. The minus sign is there because the flow *out* of the volume *decreases* the mass inside.