MA3609 Fluid Dynamics: Ship Wakes

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August 19, 2010

This presentation uses sound and automatic page transitions.

For sound you will obviously need speakers, headphones or earphones. Using Acrobat Reader under Windows seems to work fine. With Linux you will need Acrobat Reader and RealPlayer installed. I have no knowledge of Apple computers.

For the automatic transitions you will probably need to be in Full Screen Mode.

Note: On pages with animations the automatic page advancing may not work. You may have to do it yourself.

Introduction

The object of this presentation is to develop an understanding of how the difference between the phase and group velocities gives rise to the pattern of waves behind a ship.

We will look at linear waves — the amplitude and slope of the waves are small.

We also assume that the wave crest curvature is small.

Basics

If we have a wave given by

$$\eta(x, t) = \epsilon \cos(kx - \omega t)$$

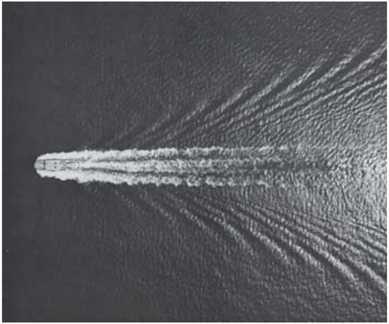
then the crests move to the right with a speed called the **phase velocity**, given by

$$c_p = \omega / k$$

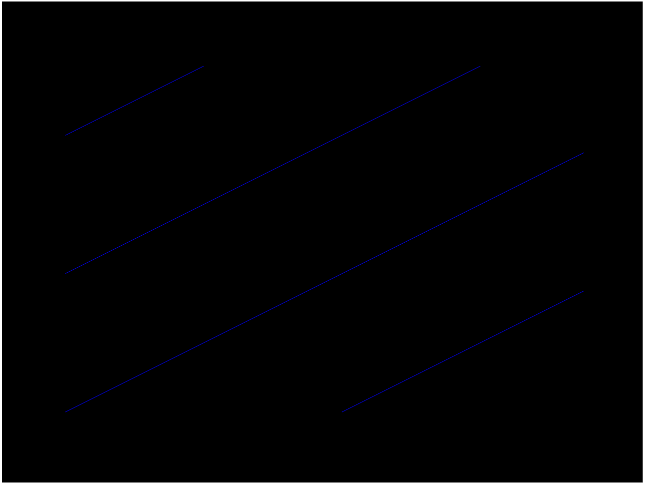
were ω is the frequency and k the wavenumber of the waves, while the energy moves with the **group velocity**

$$c_g = \frac{d\omega}{dk}$$

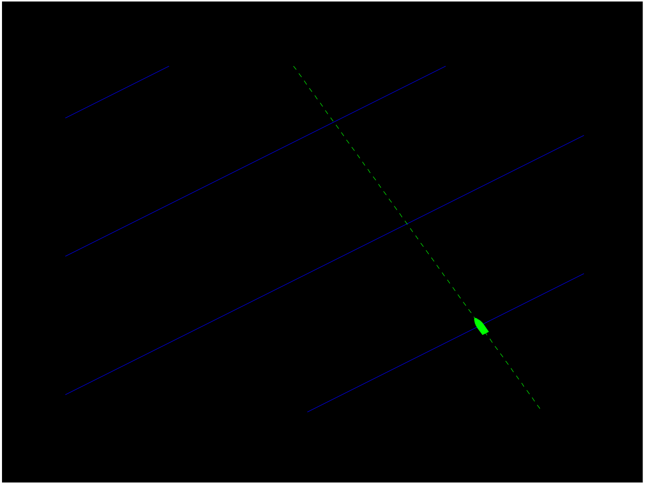
We are looking for a pattern of waves that looks fixed from the perspective of the moving boat.



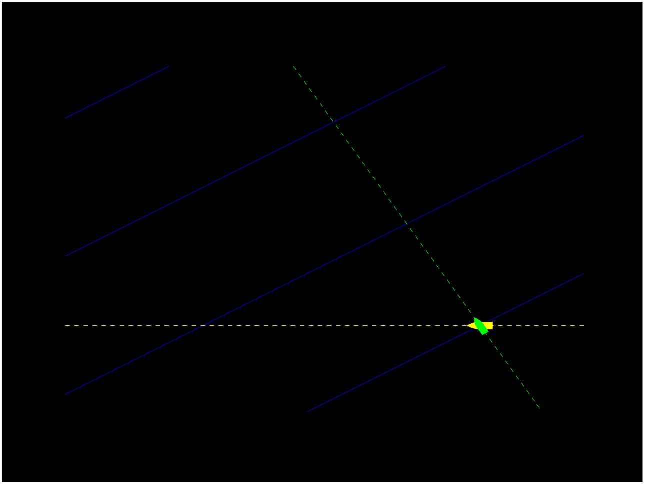
Firstly, we consider a uniform set of waves:

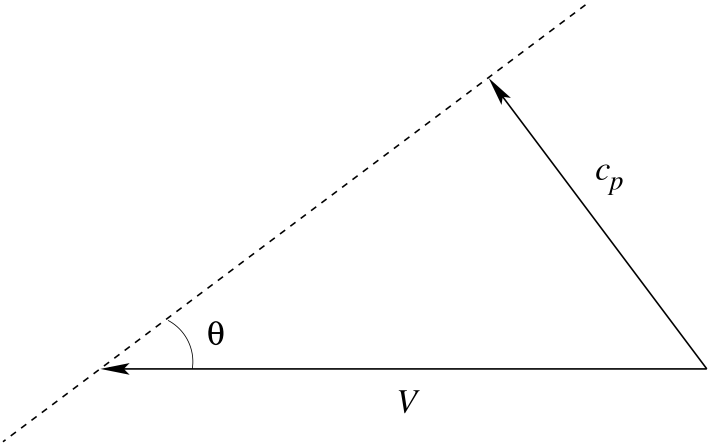


Ship moves with the waves:

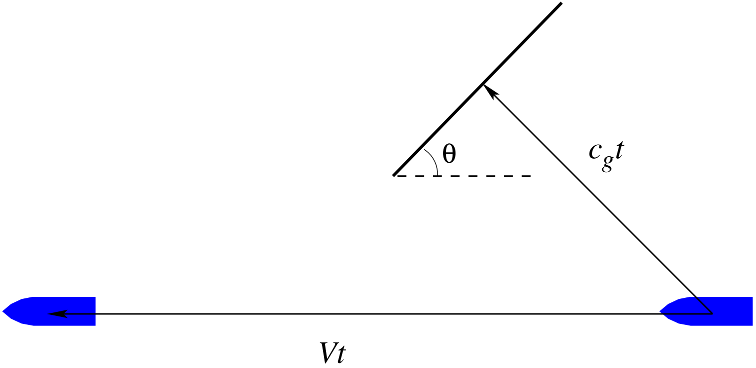


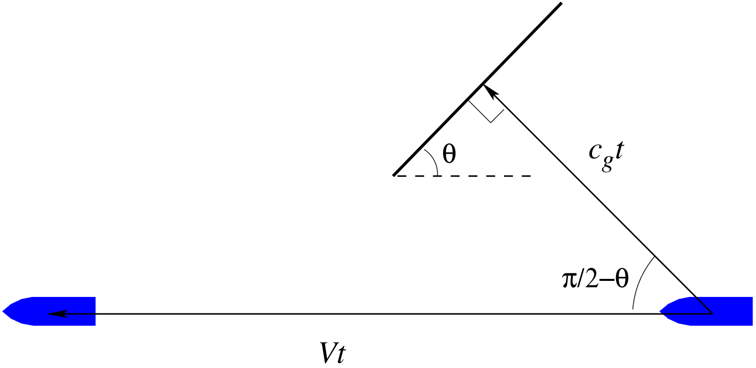
Another ship moves to the left:

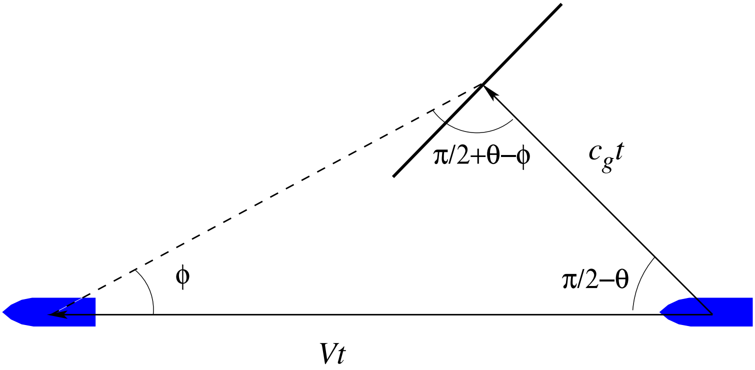




$$\frac{c_p}{V} = \sin\theta$$


θ
$c_g t$
Vt


θ
$c_g t$
π/2−θ
Vt



$$\frac{c_g t}{\sin\phi} = \frac{Vt}{\sin(\pi/2+\theta-\phi)}$$

From the geometry of wakes we have

$$\frac{c_p}{V} = \sin\theta$$

$$\frac{c_g t}{\sin\phi} = \frac{Vt}{\sin(\pi/2 + \theta - \phi)}$$

We also have for water waves on deep water

$$c_g = \frac{1}{2} c_p$$

$$\sin(\pi/2+\theta-\phi)\sin\theta = 2\sin\phi$$

Using $\cos(A-B)-\cos(A+B) = 2\sin A \sin B$ gives

$$\frac{1}{2}(\cos((\pi/2+\theta-\phi)-\theta) - \cos((\pi/2+\theta-\phi)+\theta) = 2\sin\phi$$

Simplifying

$$\frac{1}{2}(\cos(\pi/2-\phi)-\cos(\pi/2+2\theta-\phi) = \frac{1}{2}(\sin\phi+\sin(2\theta-\phi)) = 2\sin\phi$$

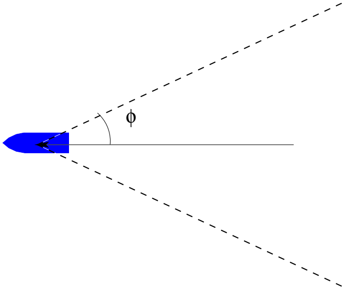
Rearranging gives

$$\sin(2\theta-\phi) = 3\sin\phi$$

$$\sin(2\theta - \phi) = 3\sin\phi$$

Firstly we conclude

$$\sin\phi \le \frac{1}{3} \qquad \Rightarrow \qquad \phi \le 0.108\pi$$

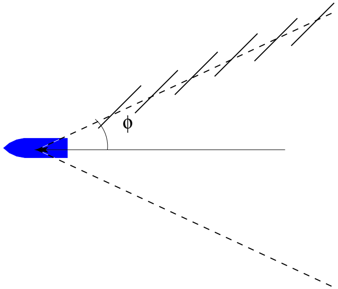


When $\sin\phi = 1/3$, $\sin(2\theta - \phi) = 1$ and so

$$2\theta - \phi = \pi/2$$

giving

$$\theta = 0.304\pi$$



For $0 < \sin\phi < 1/3$ we have two possible solutions for $2\theta - \phi$, with either

$$2\theta - \phi < \pi/2 \qquad \text{or} \qquad 2\theta - \phi > \pi/2$$